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Deleuze, Ontology, and Mathematics

Elizabeth de Freitas
Manchester Metropolitan University, Manchester, UK

Synonyms

Difference; Problematics; Intensity; Ordinality; Chance; Number; Singularity; Fractal

Introduction

Deleuze argued that we should look to the *practice* of mathematics for how to think about life more generally, advocating for a "problematics" that was strongly modeled on particular kinds of mathematical activity (Duffy 2013). The differential calculus and other kinds of mathematics are used by Deleuze to develop a philosophy of difference and repetition. He takes up mathematical concepts from analysis, topology, and algebra, citing the work of mathematicians Gottfried Leibniz, Joseph-Louis Lagrange, Richard Dedekind, Emile Galois, Henri Poincaré, Józef Wroński, and post-Kantian philosophers who wrote about mathematics, such as Salomon Maimon, Jean Bordas-Demoulin, Albert Lautman, and Jean Cavaillès.

In this entry, I focus on the role that mathematics plays in his political ontology, as developed in *Difference and Repetition* (1994) and show how these ideas were further elaborated in later work, including *A Thousand Plateaus: Capitalism and Schizophrenia* (1987), coauthored with Felix Guattari.

Problematics

Following Albert Lautman, Deleuze (1994) suggests that mathematics operates according to a problematics where "a problem has three aspects: its difference in kind from solutions; its transcendence in relation to the solutions that it engenders on the basis of its own determinant conditions; and its immanence in solutions which cover it, the problem being the better resolved the more *it is* determined." (Deleuze 1994, p. 179). Mathematical theories – like the differential calculus – are solutions to these generative problems. Thus specific mathematical theories – whatever they may be – belong to mathematics, but they are also *examples* of a "mathesis universalis" insofar as there is a universal problematics (Deleuze 1994, p. 181).

In other words, mathematics is not metaphorical for Deleuze, but is rather an excellent example for studying how this generative problematics is at work in the world (Duffy 2006). The differential calculus is *one* example of a problematics, and Deleuze also points to other areas of mathematics as perhaps even better examples, in particular the group theory of Abel and Galois. Indeed, the development of abstract algebra in the nineteenth

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century, based on the work of Abel and Galois, is declared by Deleuze to be "a radical reversal in the problem-solution relation, a more considerable revolution than the Copernican . . . For as Georges Verriest remarks, the group of an equation does not characterize at a given moment what we know about its roots, but the objectivity of what we do not know about them" (Deleuze 1994, p. 180).

But despite this gesture toward algebra, it is the differential calculus that Deleuze revisits again and again in various texts, suggesting that it gets as close as we have ever been to a differential ontology of the virtual. He synthesizes three previous commentaries on the significance of the calculus, combining them into a comprehensive theory of how the virtual and the actual are enmeshed. First, Bordas-Demoulin suggests that *dx* is the "problematic idea" and the "undetermined," second Maimon stresses how there is a reciprocal determination *dy/dx* at work rather than an individual determination, and third Wroński emphasizes the "complete determination" that is achieved in mathematical integration, where integration entails the creation of a function that is not *entailed* in the summation and thus captures the *genetic power* of differen*t/c*iation. These three key aspects of Deleuze's philosophy of difference (the undetermined, reciprocal determination, the potential) are derived from the calculus and are crucial for how Deleuze moves away from Kant's "conditions" of experience, toward a theory of how the new comes into the world.

Deleuze (1994) aligns the threesome of the "undetermined, the determinable, and the determined" with "quantitability, qualitability, and potentiality" (p. 176). Of note is the fact that these are all "abilities" or capacities that inhere in the world and that "quantitability" describes not the standard use of quantity (as that which counts and measures) but a virtual dimension of matter. The differential calculus synthesizes this three-pronged approach and becomes a powerful problematics, and by studying this problematics, we can begin to imagine how problematics might flourish elsewhere in other fields, and we can seek the "differential and genetic element" that fuels all of becoming (Deleuze 1999, p. 51). In *The Fold*, Deleuze (1993) will draw extensively on Leibniz and "Baroque mathematics" to argue for a differential folding universe, bringing together the differential calculus with a monadology. The differential relation (*dy/dx*) is the "quantitative determination" of this relational ontology of forces, be they molecular or affective, which fold the fabric of life in varying ways (de Freitas 2016). We must, says Deleuze, follow the *intrinsic quantitative relationships* that are within the qualitative "unity" of becoming. The mathematical concept of the differential is that indeterminate vibration or virtual dimension that always troubles the fixity of any quantity or object (x + dx).

Axiomatics

In a mathematical problematics, concepts quiver with indeterminacy. The square, for instance, *is* the material process of quadrature; the circle *is* the differential forces that sustain or produce or determine it. Any curve determined by an equation or static definition can be reconsidered as a dynamic machine – comprising the differential threesome of "undetermined, the determinable, and the determined." The visible circle drawn on a blackboard is simply the effect of a series of differentials of higher and higher degree. Deleuze will compare the general equation of a circle $x^2 + y^2 - R = 0$ with the differential equation $ydy + xdx = 0$ which captures "the universal of the circumference or of the corresponding function." (Deleuze 1988, p. 171). This differential equation quivers with its universality and its indeterminacy. A problematics is thus an approach to mathematics that reanimates the figures and equations of mathematics and sets them in motion, embracing the event nature of concepts, so that we might resist the tendency to imagine transcendent references for them.

Deleuze and Guattari (1987) develop this idea further and show how there has always been at least two currents in the history of mathematics, on the one hand problematics and on the other axiomatics. They compare state-sanctioned or "royal" or "major" mathematics with other lineages of mathematics, which they call "nomadic" or "minor." They describe how influential state-sanctioned traditions like the Bourbaki were intent on eliminating the dynamic and "less rigorous" mathematics of the infinitesimal calculus. Historians often describe this period as the "rigorization of mathematics," but Deleuze and Guattari (1987) will counter this narrative with an account of how state mathematics became a kind of *axiomatics* (versus *problematics*) that denied the event nature of mathematical activity (Smith 2005, 2006). State mathematics, according to this account, imposed static rules on notions such as "becoming, heterogeneity, infinitesimal, passage to the limit, [and] continuous variation" (Deleuze and Guattari 1987, p. 363).

In the context of education, this contrast between axiomatics and problematics finds a parallel in the mathematics curriculum, in the tension between the logical and ontological dimensions of mathematical activity. Mathematics curriculum is built around an axiomatic image of mathematics. That image rests on a logicist bias that creeped into the curriculum and came to dominate our understanding of mathematics. Here the term *logicism* refers to the philosophical position that mathematics can and should be reduced to logic. Of course mathematics entails (or can entail) some degree of logical deduction, but there is more to the activity of mathematics than logic and axiomatic deduction. Proof is never only deductive or inductive, but is also generative and constructive. The history of mathematics shows us how particular mathematical practices have been silenced and demoted and how the coupling of mathematics with logic became entrenched in the state-sanctioned discipline (see Hacking 2014, for excellent insights on this historical development).

Deleuze and Guattari (1987) will define state mathematics as the axiomatic deduction of properties from a set of given concepts. In contrast, minor mathematics refers to a more speculative and "less rigorous" mathematical practice that is always tamed by the epsilon-delta police of state mathematics. (Minor mathematics also refers to that side of mathematics that entails making monsters and paradoxes, a topic to which I return at the end of this entry.) In pragmatic terms, this means that minor or nomadic mathematics dwells in the mathematical concept *as an event,* while state mathematics treats the concept as a representation of essence.

Intensity and Number

Deleuze draws extensively on the French philosopher Henri Bergson to develop his ideas on differen*t/c*iation: "We will see that one of Bergson's most curious ideas is that difference itself has a number, a virtual number, a sort of numbering number." (Deleuze 1999, p. 44). The numbering number refers to the virtual intensity of number – that is, it refers to number before it is tamed by extension. Although we associate quantity with the act of measuring and partitioning extension, Deleuze uses the concept of the numbering number to get at the "intensive" source by which number mutates and resists current measures. Reclaiming number as implicated in his ontology is crucial if Deleuze, following Bergson, is to develop a radically new philosophy of difference. For if quantity is conceived within the usual quantitative paradigm, then it is always *external* to difference – an external metric or count imposed on the qualitative. But why should quantity be the one concept that escapes potentiality? Why shouldn't quantity also have an unscripted future and virtual dimension? This is precisely why Deleuze (1994) suggests that there is a "quantitability" that inheres in the virtual, a kind of inexact calculation whereby "the real in the world [is] understood in terms of fractional or even incommensurable numbers" (p. 222).

Deleuze (1994) argues that there is a virtual and actual side to number and that these two "sides" correspond to intensive and extensive relations: "every number is originally *intensive and vectorial* in so far as it implies a difference of quantity which cannot properly be cancelled, but *extensive and scalar* in so far as it cancels this difference on another plane that it creates and on which it is explicated" (p. 322, *my emphasis*). A number that cannot be "properly canceled" is a number that escapes the operators that operate on it – a number for which one cannot find an inverse that might cancel it. This is an ontological claim about the potentiality of number, a claim that forces us to rethink what terms like scalar, vectorial, and dynamic might mean in this case. According to Deleuze, number partakes in an *intensive* quantity before it is tamed and encapsulated in an *extensive* quantity. This first intensive number is not yet calibrated in terms of a fixed unit and not yet a number we can use as a counter, *and yet this virtual number is as much a part of matter as extensive magnitude*. Intensity is a crucial concept for Deleuze precisely because it points to how quantity or quanta cannot be reduced to the concept of extension, but none the less inheres in matter. Note that this is not a platonic philosophy of mathematics (in which number is an abstract ideal that transcends matter), but a philosophy of immanence, where *number is implicated in a vibrant and indeterminate matter*. De Freitas (2013) and de Freitas and Sinclair (2014) explore the implications of this ontology in relation to mathematical activity of all kinds, from recreational to expert.

Ordinality

For Deleuze (1994), the concept of ordinality, rather than cardinality, is associated with the intensity of number: "Even the simplest type of number displays this duality: natural numbers are first ordinal, in other words, originally intensive. Cardinal numbers result from these and are presented as the explication of the ordinal" (Deleuze 1994, p. 322). He aims to rescue and reuse the concept of ordinality from being only the repetition of the same unit or the fixed ordering of a set. The ordinal for Deleuze taps into the genetic differential that is the engine of his intensive ontology. Ordinality is thus a broad concept, more complex than what we tend to assume, because it plugs into the virtual depth of the "intensive spatium" (Deleuze 1994, p. 323).

Deleuze and Guattari (1987) will further develop this idea of ordinality and link it to Bergson's notion of the "numbering number":

> The Numbering Number, in other words, autonomous, arithmetic, organization, implies neither a superior degree of abstraction nor very large quantities . . . These numbers appear as soon as one distributes something in space, instead of dividing up space or distributing space itself . . . *The number is no longer a means of counting or measuring but of moving: it is the number itself that moves through space . . . The numbering number is rhythmic, not harmonic* (Deleuze and Guattari 1987, pp. 389–390, *our emphasis*).

When number becomes "the numbering number," it becomes a "mobile occupant," "ambulant fire," and the "directional number" all of which are different ways to think differently about *ordinality*. Deleuze and Guattari (1987) suggest that if we grasp or pursue the intensive nature of ordinality, we can begin to imagine how number might not only serve the control state, but instead achieve a *mobile occupying* without *whole* number counting or striating of space. It would be this other kind of smooth space where creativity (and anarchy!) can thrive. State striation will be an approach to number that is "exclusively cardinal in character," while the "ordinal, directional, nomadic, articulated number, the numbering number" produces degrees of freedom in an unstriated (or non-partitioned) space (Deleuze and Guattari 1987, p. 535). This number is ordinal in terms of how it brings forth the unscripted new, with each "count," rather than establishing the size or metric of a set.

Deleuze and Guattari (1987) use the distinction between ordinal and cardinal to discuss the political mis/uses of number, showing how different uses of number reflect very different kinds of politics. Their work looks for how state-sanctioned mathematics loses touch with the dynamism and intensity of the ordinal number. Ordinality comes to be associated with a nomadic arithmetic that mobilizes the unruly power of number to bring forth the new and to curl number into itself in varying ways. In other words, nomadic arithmetic has an ambulant, mobile dimension that contrasts with "geometry," where geometry is the practice of measuring or *striating* a static space. If the ordinal can tap into its power and plug into the intensive potentiality of the spatium (and not simply the repetition of the same), then there is possibility of creative counter-uses of number.

Revolutionary Arithmetic

Geometry and arithmetic reflect very different political relationships with territory and the distribution of power. Arithmetic always marks a certain nomadic relation to territory, because "algebra and arithmetic arise in a strongly nomad influenced world" (p. 388). They argue that arithmetic has a revolutionary potential precisely because it troubles the striating line of enclosure and control, the geometric line that measures and contains. While geometric measurement lends itself to the control of space, there is also an "independence or autonomy of the [arithmetic] Number" that subverts this kind of spatial striation (Deleuze and Guattari 1987, p. 389).

One might – quite legitimately – ask how on earth it is possible to pursue an arithmetic in this mad revolutionary way. It's hard to imagine what sort of "revolutionary" arithmetic Deleuze and Guattari have in mind, since arithmetic seems like arithmetic, no matter how you slice it. But it is possible to look into the history of mathematics and find revolutionary kinds of numbers that shook the foundation of society. In particular, the infinitesimal played such a role, being banned by the Jesuits in the 1600s for its "blasphemous" nature and outlawed in schools across Europe (de Freitas in press). For Deleuze and Guattari, the infinitesimal was one example of a radical new way of calculating, a new kind of number that produced a rich array of new mathematical ideas and techniques. But most importantly, the early use of the infinitesimal was *paradoxical*, and it was these various paradoxes that Torricelli would deploy so brilliantly in all his calculations.

The point is that the history of number supplies examples of how number can, on occasion, plug into its intensive dimension and become paradoxical (rather than orthodox). But what about today? What might the "numbering number" be today? It would have to be something that was fought over, contested, debated, denied existence, and something that seemed to shake the very foundations of mathematics. And it has to, in some way, leverage the paradoxical and the intensity of the virtual. It would have to delve into the depth of ordinal intensity and plug into the virtual potentiality of number more generally. Although such ground-moving moments in mathematics are rare, the ramifications are world changing (Alexander 2014). But perhaps there is always a volcanic rumbling in even the simplest use of number. Might there be a way to sense this rumbling in recreational mathematics, or everyday mathematics, be it practical and accurate or off-the-wall and incoherent? Might we wish to encourage more mad paradoxical arithmetic in classrooms so that the numbering number might erupt in new revolutionary garb? For Deleuze and Guattari (1987), ordinality is a generative force, a zigzagging line of flight that can furnish paradoxical but creative misuses of number. By breaking up the orthodoxy around number sense and revealing the bias toward the cardinality concept, we can begin to imagine otherwise.

Chance and Dynamic Systems

Not only does Deleuze's work help us think differently about the spatiotemporal materiality of mathematical concepts like number, it also puts forward a new commingling of chance and algorithm well suited to our digital times. Deleuze turns to the mathematics of chance and dynamic systems in order to propose new ways of thinking interaction and sociality. He also turns to topology as a powerful way of reconceiving political relationships of proximity and circulation, building on the foundation of the differential calculus. He uses key topological concepts – manifolds, curvature, inflection points, and singularities – to discuss the way that socioeconomic forces move across an event, contracting into singular points of power (singularities) that structure the behavior of those around them (see especially Deleuze 1993 and later work with Guattari).

Deleuze (1988) claims that counting is not only an act of determination, but must actually entail a kind of dice throw. For instance, as I lean in to count the faces on a polyhedron, I move from face to face (one, two, three, etc.) and this activity entails my affirming not just the individuated number of faces, but *all of numeracy*, because numeracy is continuously thread into the folding fabric of life. Counting is both a blocking of that continuity (performing a particular count and number) and an affirmation of it (feeling the flow of an absolute infinite count – all of indeterminate number) (Deleuze 1988). Deleuze inserts chance and multiplicity into each and every individual count, but also, and perhaps more controversially, he claims that number is precisely how chance thrives:

> The thrown dice form the number which brings the dice throw back. Bringing the dice throw back the number puts chance back into the fire, it maintains the fire which reheats chance. This is because number is being, unity and necessity, but unity affirmed of multiplicity as such, being which is affirmed of becoming as such. Number is present in chance in the same way as being and law are present in becoming (Deleuze 2006, pp. 29–30).

There is a reciprocity between number and chance, insofar as the "thrown dice" forms or determines a number, but the number sustains the element of chance within it. This is what subverts statistical prediction and at the same time invites new speculative methods that deploy the quantitative. Deleuze is offering us a new vision of the relationship between the qualitative and the quantitative, a new way of thinking about the role of chance in our lives. He suggests that this new relationship demands a "qualitative probabilism" (Deleuze 1988).

This suggestion may sound too well suited to our new digital culture of probabilistic reasoning and calculated publics. But this idea of the count as that which *affirms all of chance at once* is precisely why his theory is so relevant to our computational times. We need to ask how one might think the quantitative in research methods and social theory without it being just a statistical striating of space (de Freitas et al, in press). Can we imagine a future where the quantitative in education research would offer creative and ethical adventures in thought? Could such a future reconfigure the relationship between the qualitative and the quantitative? How can social theory put the quantitative to work without it simply serving the control society? The numbering number is a kind of arithmetic that is "distributing number in space, instead of dividing up space," and thus it evokes a moving number, a number that is scattered and demolished, reassembled with chance, and thrown back into the fire. It is in this way that number becomes the "mobile occupant," "directional number," and "the ambulant fire." This emphasis on number as *ambulant fire* in Deleuze's books with Guattari raises the specter of a fractal geometry of singularities.

Singularities and Fractal Monsters

In *A thousand Plateaus*, in the chapter on smooth and striated space, Deleuze and Guattari (1987) mention the Koch snowflake and the Sierpensky sponge as examples of smooth spaces insofar as these kinds of figures pursue a fractional dimension, somewhere between line and plane or between plane and solid. Any space with a fractional dimension escapes conventional measures and is "the index of a properly directional space." In other words, the dimensionality of a smooth space is determined by that which moves through it, rather than by some magnitude of containment. These fractal examples show how ordinality becomes inflected by chance, recursion, and singularity. They also point to a new kind of computational becoming, for it is through the iterative mobile calculating of a space-filling fractal that a line can fill a plane without ceasing to be a line.

For Deleuze and Guattari (1987), this is a crucial aspect of creative and dynamic spaces – that they do not have "a dimension higher than that which moves through it or is inscribed in it." (p. 488). This is how number resists the containment of the cardinal, which is always the count of *how many* rather than the *flow* of the ordinal. Indeed, a condition for smooth space will be a mobile ordinal numbering, *occupying* without whole number counting. Fractals pursue this recursive directional number – indeed, monstrous calculations of this kind are another example of how calculation becomes monstrous and revolutionary (de Freitas 2015). Massumi (1992) declares "The plane of life itself . . . is a space-filling fractal of infinite dimension." (p. 23). The monstrous nondifferentiable but continuous Koch snowflake fractal, for example, proliferates singularities as it repeats and expands. Deleuze and Guattari use this example to describe a fractal image of life, to show how calculation can be *machinic but non-axiomatic*. They use fractals as monstrous calculating devices that transform the concept of measure and multiplicity.

The concept of the fractal became increasingly important in Guattari's writing on chaosmosis, where various processes of fractalization figure prominently in thinking the sociopolitical subject. Fractals occupy fractional dimensional spaces and thereby break with conventions regarding space and embodiment. A fractal recombinant subject no longer abides by the dominant image of the organism and phenomenological subjectivity. Chance and algorithm commingle in the fractal subject, mutating and folding universe in new ways. A fractal subjectivity will find very different ways of becoming in the contemporary world. We can see that the differential calculus remains pivotal in the later work of Deleuze and Guattari, but it is the concept of the singularity that takes on more significance, doing the *structuring* work that is needed for fractal folding and recombinant becoming. Deleuze and Guattari (1987) tap into the infinitesimal as the calculating engine of their ontology, a means of differen*t/c*iating the mathematical continuum (achieving the necessary "reciprocal presupposition" of our "common matter" (pp. 108–109)). This later work with Guattari builds on the early work of Deleuze, further developing the mathematical concept of singularity – as the generative and immanent dark precursor to life itself – to address the fractal folds of computational culture and post-quantum subjectivity.

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